A MATHEMATICAL MODELLING OF INCONSTANT TEMPERATURE MODULATION IN HUMAN BODY DUE TO ARTERIAL BLOOD TEMPERATURE

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ABSTRACT

The modeling of temperature modulation in human body has immense significance to understand the physiology of the human body. The outer surface of skin is assumed to be exposed to environment temperature. Any disturbance in the temperature regulation may cause lots of abnormality in the body. In the present paper an attempt has been made to inconstant temperature modulation in dermal region of human body by changing arterial blood temperature for transient case. The nodal temperatures are noted at different time intervals.

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1. Introduction

Mathematical models have tremendous potential for advancing the understanding of the physical processes involved in medical science and biological processes etc. Body temperature plays an important role to regulate every biological system. Metabolism is main source to generate heat continuously within the human body. If this heat is not lost from the body, then the temperature of body will keep on rising continuously. Thus the body heat is continuously lost to the surrounding. The process which maintains thermal balance between the body and surrounding is known as thermal regulation system of the body.

There are many factors affecting body temperature like atmospheric temperature, the body mass and surface area of the body, Human body is composed of three layers namely sub dermis, dermis and dermis. Due to absence of blood vessels in epidermis, the blood circulation in epidermis is negligible. And dermis blood flow is variable while uniform in subcutaneous tissue. The rate of blood flow in SST region is most variable in comparison to other parts of the body.Many investigations have been made to study thermal responses in normal and abnormal conditions. Perl (1962) combined differential forms of Flick's perfection principle with heat conduction and matter diffusion equations and metabolic term to obtain equation. Perl and Hirsch (1966) used this equation to test the transient response for measuring local tissue blood flow on dog and rabbit kidney. Trezek and Cooper (1968) computed thermal conductivity of tissue by taking all parameters as constant.Cooper and trezek (1972, a, b) obtain solution of equation in SST region by taking all parameters as constant. Patterson (1976, 1978) made experimental attempts, to determine temperature profiles in skin and subcutaneous region. Saxena (1978) solved equation by similarity transformation in SST region. Saxena and Arya (1981) used variation finite element methods to solve the problem of steady state temperature distribution in three layered skin and subcutaneous region. Saxena, Arya and Bindra (1987) obtained unsteady state temperature distribution in human skin and subcutaneous tissue region, by using the variation finite method and Laplace transform method. Saxena and Pardasani (1991) discussed the effect of tumor on temperature distribution in human skin. Yadav (1998) solved various types of temperature distribution problem in skin and SST region with thermal injury.

Although in this field lot of work has been done by many mathematician, scientists and other researchers but it is not possible to explain the work of everyone .Results obtained by all mentioned mathematicians, scientists and researchers are much under consideration, but their studies are confined to take constant or average values of arterial blood temperature and venous blood temperature is equal to tissue temperature, which are practically not possible in the skin and SST region.In the present paper we have taken arterial blood temperature as position dependent and venous blood temperature is also taken variable.

2. Statement of the Problem

Perl's Bio heat partial differential equation in two dimensional transient state case for heat distribution in the tissues of SST region of human body can be written as:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + m_b c_b (T_b - T) + S = \rho c \frac{\partial T}{\partial t} \qquad (2.1)$$

Here the effect of metabolic heat generation and blood mass flow are given by the terms S and $m_b c_b(T_b\text{-}T)$ respectively. T_b, K, ρ, c, m_b and $c_b$are body core temperature, thermal conductivity, density and specific heat of tissue; blood mass flow rate and specific heat of blood respectively. Right hand side of eq. (2.1) shows the storage of heat in tissues. The first two terms of the left hand side represents conduction of heat in the tissues, caused by the temperature gradient and third term is for heat transport between the tissues and microcirculatory blood perfusion. The last term represent heat generation due to metabolism. Using (2.1) a mathematical model for heat flow in living tissues for one dimensional unsteady state case can be written as

3. Boundary conditions: The outer surface of the body is exposed to the environment and heat loss at this surface takes place due to conduction, convection, radiation and evaporation. Thus the boundary conditions at

the outer surface

$$-k\frac{\partial y}{\partial x} = h(T - T_a) + LE, for\ t > 0 \qquad 2.2$$

Where h heat transfer coefficient, T_a is atmospheric temperature, L and E are respectively, the latent heat and rate of evaporation and $\frac{\partial t}{\partial n}$ is the partial derivatives of T along the normal to the skin surface.

the inner surface

$$T_V = qT \qquad 2.3$$

Where q is known constant its value is nearer to 1.

4. Initial condition:

The following additional initial and boundary conditions are also used;

$$T(x,0) = 22.87 + px, p > 0\ , 0 \le x \le a_3$$
$$T(a_3,t) = T_b$$
$$T(a_3,0) = T_b$$

Where p is a known constant.

5. Solution

If I_1, I_2 and I_3 are the values of I in three sub-regions then

$$I = \sum_{i=1}^{3} I_i \ .$$

Now using equation (2.1) ,(2.2)with equation (2.3), we get

$$I_1 = \frac{K_1}{2a}(T_1 - T_0)^2 + \frac{h}{2}(T_0 - T_a)^2 + LET_0$$

$$I_2 = (T_2 - T_1)^2 E_1 + (T_2 a_1 - T_1 a_2)^2 E_2 + (T_2 a_1 - T_1 a_2)(T_2 - T_1)E_3 + (T_2 a_1 - T_1 a_2)E_4 + (T_2 - T_1)E_5 + E_6$$

$$I_3 = (T_3 - T_2)^2 F_1 + (a_3T_2 - a_2T_3)^2 F_2 + (T_3 - T_2)F_3 + (a_3T_2 - a_2T_3)F_4 + (T_3 - T_2)(a_3T_2 - a_2T_3)F_5 + F_6$$

Where,

$$E_1 = \frac{a_2K_1 - a_1K_3}{2(a_2 - a_1)^2} + \frac{K_3 - K_1}{(a_2 - a_1)^2}\left(\frac{a_1 + a_2}{4}\right) + \frac{q^2M(3a_2^2 + 2a_2a_1 + a_1^2)}{24(a_2 - a_1)},$$

$$E_2 = Mq^2 \frac{1}{4(a_2 - a_1)},$$

$$E_3 = -\frac{MqT_b(a_2 - a_1)(a_1 + 3a_2)}{12(a_3 - a_1)},$$

$$E_4 = -mq\frac{T_b(a_2 - a_1)}{3(a_3 - a_1)} - \frac{S}{2},$$

$$E_5 = mq^2\frac{(a_1 + 2a_2)}{6(a_2 - a_1)},$$

$$E_6 = M\left(\frac{T_b}{a_3 - a_1}\right)^2 \frac{(a_2 - a_1)^3}{8},$$

$$F_1 = \frac{q^2M}{6}\frac{(a_3^2 + a_2^2 + a_3a_2)}{(a_3 - a_2)} + \frac{K_3}{2(a_3 - a_2)},$$

$$F_2 = Mq^2 \frac{1}{2(a_3 - a_2)}$$

$$F_3 = -\frac{T_bqM(2a_2^2 + 3a_3^2 + 2a_2a_3 - 3a_1a_3 - 3a_1a_2)}{6(a_3 - a_1)} - \frac{S(a_3 + a_2)}{2},$$

$$F_4 = -\frac{T_bqm}{2}\frac{(a_3 + a_2 - 2a_1)}{(a_3 - a_1)} - S,$$

$$F_5 = \frac{mq^2 (a_3 + a_2)}{2(a_3 - a_2)},$$

$$F_6 = M\left(\frac{T_b}{a_3 - a_1}\right)^2 \frac{(a_3^2 + a_2^2 + a_3 a_2 - 3a_1 a_2 - 3a_1 a_3 + 3a_1^2)^3 (a_3 - a_2)}{6},$$

And $M = m_b C_b$

Since T_3 is equal to body core temperature, so we minimize I for T_0 , T_1 and T_2. Accordingly we get following system of equations:-

$$L_1 T_0 - L_2 T_1 = W_0; \quad M_1 T_0 + M_2 T_1 + M_3 T_2 = W_1 ; \quad N_1 T_1 + N_2 T_2 = W_2$$

Where,

$$L_1 = \frac{K_1}{a_1} + h$$

$$L_2 = -\frac{K_1}{a_1}$$

$$W_0 = (hT_a - LE)$$

$$M_1 = -\frac{K_1}{a_1}$$

$$M_2 = \frac{K_1}{a_1} + 2E_1 + 2a_2^2 E_2 - 2a_2 E_5$$

$$M_3 = -2E_1 - 2a_1 a_2 E_2 + (a_1 + a_2)E_5$$

$$W_1 = E_3 - a_2 E_4$$

$$N_1 = -2E_1 - 2a_1 a_2 E_2 + (a_1 + a_2)E_5$$

$$N_2 = 2E_1 + 2a_1^2 E_2 - 2a_1 E_5 + 2F_1 + 2a_3^2 F_2 - 2a_3 F_5$$

$$W_2 = \{2F_1 + 2a_2 a_3 F_2 - (a_2 + a_3)F_5\}T_3 - E_3 + a_1 E_4 + F_3 - a_3 F_4$$

6. Numerical Result and Discussion:

The SST region is divided into three parts namely epidermis (the outer one), dermis (under the epidermis) and sub dermis (below the dermis layer). The value of K, M, S are assumed constant epidermis layer in dermis layer. K, M, S and other values are calculated using Lagrange's interpolation polynomial. No blood vassals present in sub dermis so values of those parameters are taken zero.In present paper mathematical model has been developed to analyze the temperature variation in dermal region. The total thickness is taken 0.95cm.The thickness of subcutaneous; dermis and epidermis are 0.50cm, 0.35cm and 0.10cm respectively.

The values of physical and physiological parameters have been taken from Cooper and Trazek as given below.

Table 1: Values for physical and physiological parameter

Thermal Conductivity (cal/cm min °C)	**Heat Transfer Coefficient h (cal/cm^2 min °C)**	**Specific Heat of Tissues c (cal/gm °C)**
K_1=0.060, K_2=0.045, K_3=0.030	0.009	0.830
Blood Density of Tissues ρ (gm/cm^3)	**Latent Heat L (cal/gm)**	**Body Core Temperature T_b (°C)**
1.090	579.0	37

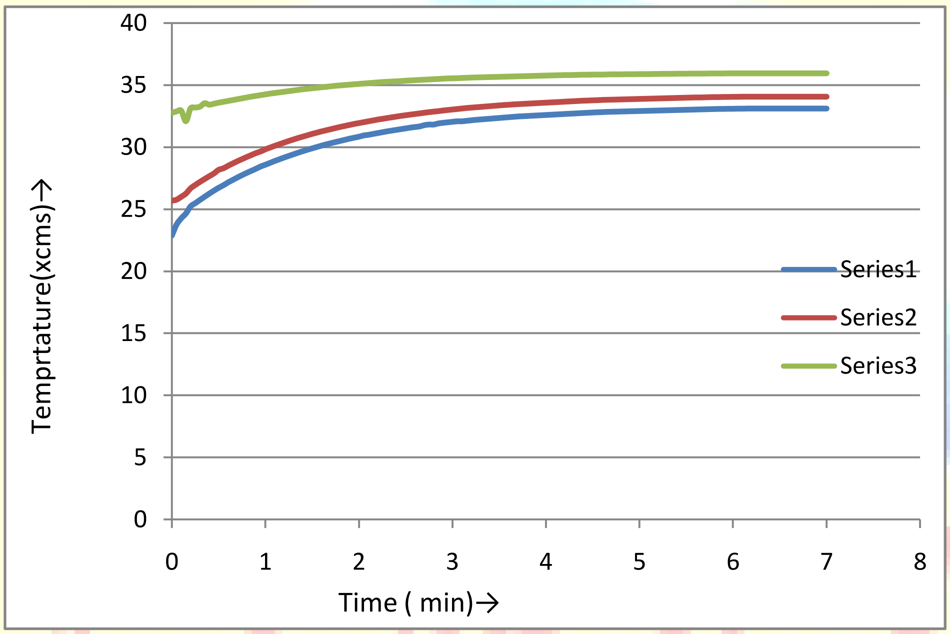
Table 2: M, S and E for different atmospheric temperature

Atmospheric Temperature T_a (°C)	**Rate of Evaporation E (gm/ cm^{2}min)**	**Blood Mass Flow Rate M (cal/ cm min. °C)**	**Rate of metabolism S (cal/cm^3 min^1)**
23	0, 0.24x10^{-3}, 0.48x10^{-3}	0.0180	0.0180
33	0.24x10^{-3},0.48x10^{-3}	0.0315	0.0180

Graphs are plotted between temperature T and distance x for different values of q and different atmospheric temperature. Numerical solutions are obtained for sub -dermis, dermis and epidermis region. Initially it is assumed that SST region is fully insulated. So temperature of each layer at time t=0 is equal to the 37^{o} C.

In the entire graph, heat loss from epidermis layer is more than that of the dermis and SST region due to evaporation. Temperature variation is seen by changing the value of q. It is observed that the fall in tissue temperature is more at same rate of evaporation and lower atmospheric temperature. Also the tissue temperature decreases with the rise in venous blood temperature.

Fig. 1 Temperature distribution in dermal region with respect to Time



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